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Collective resonances in the soliton model approach to meson-baryon scattering

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Abstract. The proper description of hadronic decays of baryon resonances has been a long-standing problem in soliton models for baryons. In this paper I present a solution to this problem in the three-flavor Skyrme model that is consistent with large-N_C consistency conditions. As an application I discuss hadronic pentaquark decays and show that predictions based on axial current matrix elements are erroneous.

PACS. 11.15.Pg Expansions for large numbers of components (*e.g.*, $1/N_c$ expansions) – 12.39.Dc Skyrmions – 13.75.Jz Kaon-baryon interactions

1 Introduction

Commonly hadronic decays of baryon resonances are described by a Yukawa interaction of the generic structure

$$\mathcal{L}_{\rm int} = g\,\bar{\psi}_{B'}\,\phi\,\psi_B\,, \qquad (1)$$

where B' is the resonance that decays into baryon B and meson ϕ and g is a coupling constant. It is crucial that this interaction Lagrangian is *linear* in the meson field.

The situation is quite different in soliton models that are based on action functionals of only meson degrees of freedom, $\Gamma = \Gamma[\Phi]$. These action functionals contain classical (static) soliton solutions, $\Phi_{\rm cl}$, that are identified as baryons. The interaction of these baryons with mesons is described by the (small) meson fluctuations about the soliton: $\Phi = \Phi_{\rm cl} + \phi$. By pure definition, we have

$$\frac{\delta\Gamma[\Phi]}{\delta\Phi}\Big|_{\Phi=\Phi_{\rm cl}} = 0\,. \qquad (2)$$

Thus, there is no term linear in ϕ to be associated with the Yukawa interaction, eq. (1). This puzzle has become famous as the Yukawa problem in soliton models. However, this does not mean that hadronic decays of resonances cannot be described in soliton models. Rather they have to be extracted from meson baryon scattering amplitudes, just as in experiment. In soliton models two-meson processes acquire contributions from the second-order term

$$\Gamma^{(2)} = \frac{1}{2}\,\phi\,\frac{\delta^2\Gamma[\Phi]}{\delta^2\Phi}\Big|_{\Phi=\Phi_{\rm cl}}\,\phi\,. \qquad (3)$$

This expansion simultaneously represents an expansion in N_C, the number of color degrees of freedom: $\Gamma = \mathcal{O}(N_C)$, $\Gamma^{(2)} = \mathcal{O}(N_C^0)$ while terms $\mathcal{O}(\phi^3)$ vanish in the limit $N_C \to \infty$. This implies that $\Gamma^{(2)}$ contains all large-N_C information about hadronic decays of resonances. We may reverse that statement to argue about computations of hadronic decay widths in soliton models: Their results and those obtained from $\Gamma^{(2)}$ *must* be identical in the limit $N_C \to \infty$. Unfortunately, the most prominent baryon resonance, the Δ, becomes degenerate with the nucleon as $N_C \to \infty$. It is stable in that limit and its decay is not subject to the above-described litmus test. The situation is more interesting when extending soliton models to flavor $SU(3)$. In the so-called rigid-rotator approach (RRA) that generates baryon states as (flavor) rotational excitations of the soliton, resonances emerge that dwell in the anti-decuplet representation of flavor $SU(3)$. The most discussed (and disputed) such state is the Θ^+ pentaquark with zero isospin and strangeness $S = +1$. In the limit $N_C \to \infty$ the anti-decuplet states maintain a non-zero mass difference with respect to the nucleon. Therefore, the decay properties of Θ^+ as predicted in any soliton model must also be seen in the S-matrix for koan-nucleon scattering as computed from $\Gamma^{(2)}$. In the $S = -1$ sector the resulting equations of motion for ϕ yield a P-wave bound state whose occupation serves to describe the ordinary hyperons, Λ, Σ, Σ^*, etc. Therefore this treatment of hyperon states is called the bound-state approach (BSA). The above-discussed litmus test requires that the BSA and RRA give identical results for the Θ^+ properties as $N_C \to \infty$. This did not seem to be true and it was argued that the prediction of pentaquarks would be a mere artifact of the RRA [1]. Here we will show that this conclusion is premature and that pentaquark states do indeed emerge in both approaches. Furthermore, the comparison between the BSA and RRA provides an unambiguous computation

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of pentaquark widths: It differs substantially from previous approaches based on assuming pertinent transition operators for $\Theta^+ \to KN$ [2,3]. Details of these studies are contained in ref. [4] and ref. [5] may be consulted for a review on $SU(3)$ soliton models.

2 Constrained fluctuations and Θ^+ width

When restricted to modes spanned by the soliton's rigid rotations, the P-wave fluctuations in strangeness direction have two bound states with eigenenergies

$$\omega_\pm = \frac{1}{2}\left[\sqrt{\omega_0^2 + \frac{3\Gamma}{2\Theta_K}} \pm \omega_0\right], \qquad (4)$$

where Θ_K is the moment of inertia for the rotation of the soliton into strangeness direction and Γ is the functional of the soliton that measures flavor symmetry breaking. The latter would be zero if the masses of the strange and non-strange quarks were equal. Both functionals are $\mathcal{O}(N_C)$. The subscript on $\omega_\pm$ refers to the strangeness quantum number of the bound state. Hence $\omega_0 = N_C/(4\Theta_K)$ removes the degeneracy between $S = \pm 1$ baryons. This contribution stems from the Wess-Zumino term in $\Gamma[\Phi]$. In accordance with the above discussion $\omega_\pm = \mathcal{O}(N_C^0)$. While ω_- is the energy of the above-mentioned bound state describing ordinary hyperons, ω_+ is eventually utilized to construct pentaquark states.

In the RRA the collective coordinates $A(t) \in SU(3)$ that parameterize the flavor orientation of the soliton are canonically quantized. The resulting Hamiltonian is (numerically) exactly diagonalized for arbitrary N_C [4] and symmetry breaking [6]. The so-computed mass difference between the states that for $N_C = 3$ correspond to the Λ (Θ^+) and the nucleon approaches ω_- (ω_+) as $N_C \to \infty$. This suggests that indeed the RRA and BSA are identical in that limit. This identity has a caveat when the restriction that BSA modes are spanned by the rigid rotation is removed. Though $\omega_- < m_K$ still corresponds to a true bound state, ω_+ is a continuum state. Thus, a pronounced resonance structure would be expected in the BSA phase shift around $\omega = \omega_+$. Unfortunately, that is not the case, as seen from fig. 1. The BSA phase shift hardly reaches $\pi/2$ rather than quickly passing through this value. The ultimate comparison requires to generalize the RRA to the rotation-vibration approach (RVA)

$$U(\boldsymbol{x},t) = A(t)\xi_0(\boldsymbol{x}) \exp\left[\frac{i}{f_\pi}\sum_{\alpha=4}^{7}\lambda_\alpha\widetilde{\eta}_\alpha(\boldsymbol{x},t)\right]\xi_0(\boldsymbol{x})\,A^\dagger(t), \qquad (5)$$

where $\xi_0(\boldsymbol{x}) = \exp[i\hat{\boldsymbol{x}}\cdot\boldsymbol{\tau}F(|\boldsymbol{x}|)/2]$ is the chiral field representation of the soliton ($\Phi_{\rm cl}$) and $A(t) \in SU(3)$ parameterizes the collective rotations. Modes that correspond to the collective rotations must be excluded from the fluctuations $\widetilde{\eta}$, *i.e.* the fluctuations must be orthogonal to the zero mode $z \sim \sin(F/2)$. Imposing the corresponding constraints for these fluctuations (and their conjugate momenta) yields integro-differential equations listed in ref. [4]. In the BSA $A(t)$ is restricted to $SU(2)$ but there is no constraint on the fluctuations η. The above-discussed litmus test requires that the scattering data computed from $\widetilde{\eta}$ and η be identical when N_C is sent to infinity.

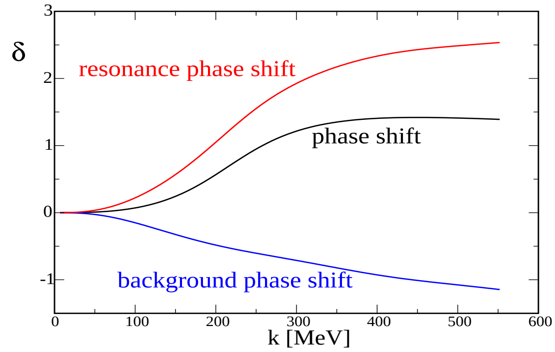

Fig. 1. (Colour on-line) Phase shift computed in the BSA (middle, black line) and the resonance phase (top, red line) shift after removal of the background (bottom, blue line) contribution in the RVA.

For the moment let us omit the coupling between $\widetilde{\eta}$ and the collective soliton excitations. This truncation defines the background wave function $\overline{\eta}$ (also orthogonal to the zero mode). Treating $\overline{\eta}$ as an harmonic fluctuation provides the background phase shift shown as the blue on-line curve in fig. 1. Remarkably, the difference between the phase shifts of $\overline{\eta}$ and η clearly exhibits a distinct resonance structure. This is the resonance phase shift to be associated with the Θ^+ pentaquark in the limit $N_C \to \infty$!

The parameterization, eq. (5) is not a solution to the classical equation of motion, The strategy rather is to solve them order by order in the N_C expansion. Hence the arguments deduced from eq. (2) do not apply and an interaction Hamiltonian that is linear in the fluctuations indeed emerges. This generates Yukawa couplings between the collective soliton excitations and the fluctuations $\widetilde{\eta}$. In ref. [4] we have derived this Hamiltonian keeping all contributions that survive as $N_C \to \infty$. The corresponding Yukawa exchanges extend the integro-differential equations for $\overline{\eta}$ by a separable potential V_Y, therewith providing the equations of motion for $\widetilde{\eta}$. For $N_C \to \infty$ the equation of motion for $\widetilde{\eta}$ is solved by $\widetilde{\eta} = \eta - \langle z|\eta\rangle z$. The phase shifts extracted from η and $\widetilde{\eta}$ are identical because $z(|\boldsymbol{x}|)$ is localized in space. Thus, the BSA and RVA yield the same spectrum and are indeed equivalent in the large-N_C limit. But, the RVA provides a distinction between resonance and background contributions to the scattering amplitude. Applying the R-matrix formalism on top of the constrained fluctuations $\overline{\eta}$ shows that V_Y *exactly* contributes the resonance phase shift shown in fig. 1 when the Yukawa coupling is computed for $N_C \to \infty$. This identifies the exchange of a state predicted in the RRA which thus is no artifact. In contrast, pentaquarks are also predicted by the BSA; just well hidden. Nevertheless, collective coordinates are mandatory to obtain finite-N_C corrections to the BSA for the properties of Θ^+. Though not all $\mathcal{O}(1/N_C)$ operators were included in ref. [4], subleading effects are substantial. For example, in the case $m_K = m_\pi$ the mass

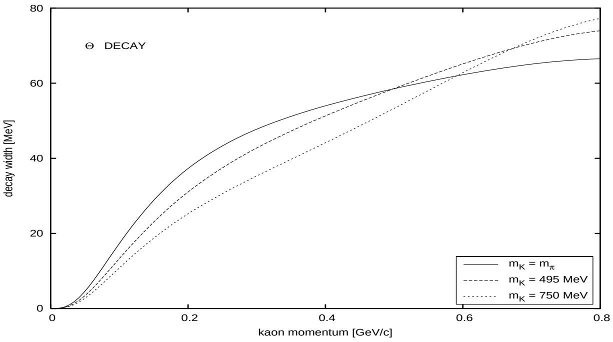

Fig. 2. Skyrme model prediction for the decay width, $\Gamma(\omega)$ of Θ^+ for $N_C = 3$ as a function of the kaon momentum $k = \sqrt{\omega^2 - m_K^2}$, cf. eq. (6). Top curve: $m_K = m_\pi$, bottom curves: $m_K \neq m_\pi$.

difference with respect to the nucleon increases by a factor two from ω_0 to $(N_C + 3)/4\Theta_K$ for $N_C = 3$.

The separable potential V_Y also yields the general expression for the width as a function of the kaon energy $\omega_k = \sqrt{k^2 + m_K^2}$ from the R-matrix formalism [4]:

$$\Gamma(\omega_k) = 2k\omega_0 \left| X_\Theta \int_0^\infty r^2 \mathrm{d}r\, z(r) 2\lambda(r) \overline{\eta}_{\omega_k}(r) + \frac{Y_\Theta}{\omega_0} \left(m_K^2 - m_\pi^2\right) \int_0^\infty r^2 \mathrm{d}r\, z(r) \overline{\eta}_{\omega_k}(r) \right|^2 . \quad (6)$$

Here $\overline{\eta}_{\omega_k}(|\boldsymbol{x}|)$ is the P-wave projection of the background wave function $\overline{\eta}$ for a prescribed energy ω_k and $\lambda(|\boldsymbol{x}|)$ is a radial function that stems from the Wess-Zumino term. The matrix elements (X_Θ and Y_Θ) of the collective coordinate operators that enter eq. (6) are computed from the eigenstates of the collective coordinate Hamiltonian. The resulting width is shown for $N_C = 3$ in fig. 2 for both the flavor symmetric case and the physical kaon-pion mass difference. As a function of momentum, there are only minor differences between these two cases. Assuming the observed resonance to be the (disputed) $\Theta^+(1540)$ a width of roughly 40 MeV is read off from fig. 2 [4]. However, fig. 1 suggests that the resonance should be about 200 MeV above threshold which corresponds to $M_{\Theta^+} \approx 1.65$ GeV. Hence it seems very unlikely that chiral soliton models predict a light and very narrow pentaquark, though the numerical results for masses and widths of pentaquarks are model dependent.

3 Conclusion

In this paper I have presented a thorough comparison [4] between the bound state (BSA) and rigid-rotator approaches (RRA) to chiral soliton models in flavor $SU(3)$. Though I have only considered the simplest such model, the actual analysis merely concerns the treatment of kaon degrees of freedom. Therefore the qualitative results are valid for *any* chiral soliton model.

A sensible comparison with the BSA requires the consideration of harmonic oscillations in the RRA as well. They are incorporated via the rotation-vibration approach (RVA), however constraints must be implemented to ensure that the introduction of such fluctuations does not double-count any degrees of freedom. The RVA clearly shows that the prediction of pentaquarks is not an artifact of the RRA, pentaquarks are genuine within chiral soliton models. Only within the RVA chiral soliton models generate interactions for hadronic decays. Technically, the derivation of this Hamiltonian is quite involved, however, the result is as simple as convincing: In the limit $N_C \to \infty$, in which the BSA is undoubtedly correct, the RVA and BSA yield identical results for the baryon spectrum and the kaon-nucleon S-matrix. This identity also holds when flavor symmetry breaking is included. This demonstrates that collective coordinate quantization may be successfully applied regardless of whether or not the respective modes are zero modes.

In the flavor symmetric case the interaction Hamiltonian contains only a *single* structure (X_Θ in eq. (6)) of $SU(3)$ matrix elements for the $\Theta^+ \to KN$ transition. Any additional $SU(3)$ structure only enters via flavor symmetry breaking. This proves earlier approaches [2,3] incorrect that adopted any possible structure that would contribute in the large-N_C limit and fitted coefficients from a variety of hadronic decays under the assumption of $SU(3)$ relations. The study presented in this talk thus suggests that it is not worthwhile to bother about the obvious arithmetic error in ref. [2] that was discovered earlier [7,8] because the conceptual deficiencies in such width calculations are more severe. Assuming $SU(3)$ relations among hadronic decays is not a valid procedure in chiral soliton models. The embedding of the classical soliton breaks $SU(3)$ and thus yields different structures for different hadronic transitions.

Even in case pentaquarks turn out not to be what some recent experiments have suggested, they have definitely been very beneficial in combining the bound state and rigid-rotator approaches and solving the Yukawa problem in the kaon sector; both long-standing puzzles in chiral soliton models.

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